Mode locking in ac+dc generalized Frenkel-Kontorov model

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1 Introduction

2 ac±dc driven Frenkel - Kontorova model
Array of coupled harmonic oscillators $u_i$, in a periodic substrate pinning potential $V(u) = \frac{K}{(2\pi)^2} [1 - \cos(2\pi u)]$. 

where $K$ is the pinning strength.
Standard Frenkel - Kontorova model

- Array of coupled harmonic oscillators $u_i$, in a periodic substrate pinning potential $V(u) = \frac{K}{(2\pi)^2}[1 - \cos(2\pi u)]$.
- where $K$ is pinning strength.
Array of coupled harmonic oscillators \( u_i \), in a periodic substrate pinning potential \( V(u) = \frac{K}{(2\pi)^2} [1 - \cos(2\pi u)] \).

where \( K \) is pinning strength.
Winding number $\omega = \frac{p}{q}$. $q$ is number of particles per unit cell.
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\( \omega = 1 \) - one particle per potential well.
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$\omega = 1$ - one particle per potential well.
Commensurate structures

- $\omega = 1/2$ - two particles per potential well
Commensurate structures

- $\omega = 1/2$ - two particles per potential well
- $\omega = 1/3$ - three particles per potential well
Commensurate structures

- $\omega = \frac{1}{2}$ - two particles per potential well
- $\omega = \frac{1}{3}$ - three particles per potential well
- $\omega = \frac{1}{4}$ - four particles per potential well
Commensurate structures

- $\omega = 1/2$ - two particles per potential well
- $\omega = 1/3$ - three particles per potential well
- $\omega = 1/4$ - four particles per potential well
- ...
1 Introduction

2 ac+dc driven Frenkel - Kontorova model
Overdamped model
Overdamped model

\[
\dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'(u) + F(t)
\]

where \( F(t) = \bar{F} + F_{ac} \cos 2\pi \nu_0 t \)
Overdamped model

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where \( F(t) = \bar{F} + F_{ac} \cos 2\pi \nu_0 t \)

\[ l = -\frac{N}{2}, \ldots, \frac{N}{2}. \]
Overdamped model

\[ \dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'(u) + F(t) \]

where \( F(t) = \bar{F} + F_{ac} \cos 2\pi \nu_0 t \)

\( l = -\frac{N}{2}, \ldots, \frac{N}{2} \).

Equation have been integrated using fourth order Runge-Kutta method.
Overdamped model

\[ \dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'(u) + F(t) \]

where \( F(t) = F_0 + F_{ac} \cos 2\pi \nu_0 t \)

\( l = -\frac{N}{2}, \ldots, \frac{N}{2} \).

Equation have been integrated using fourth order Runge-Kutta method.

No passing rule!
Overdamped model

\[ \dot{u}_I = u_{I+1} + u_{I-1} - 2u_I - V'(u) + F(t) \]

where \( F(t) = \bar{F} + F_{ac} \cos 2\pi \nu_0 t \)

\[ I = -\frac{N}{2}, \ldots, \frac{N}{2} \]

Equation have been integrated using fourth order Runge-Kutta method.

No passing rule!
Subharmonic steps are negligibly small.
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Changing the pinning strength $K$.

- Changing of pinning strength $K$. 

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![Graph showing the effect of changing pinning strength $K$.]
Changing the $F_{ac}$.

- Changing the amplitude of ac force $F_{ac}$. 
Changing the amplitude of ac force $F_{ac}$. 

![Graph showing the change in amplitude of ac force $F_{ac}$]
• Bessel like oscillations.
Bessel like oscillations.
Asymmetric deformable substrate potential

- Asymmetric deformable potential.
Asymmetric deformable potential.

\[ V(u) = \frac{K}{(2\pi)^2} \frac{(1-r^2)^2[1-\cos(2\pi u)]}{[1+r^2+2r \cos(\pi u)]^2} \]
Asymmetric deformable substrate potential

- Asymmetric deformable potential.
- \[ V(u) = \frac{K}{(2\pi)^2} \frac{(1-r^2)^2[1-\cos(2\pi u)]}{[1+r^2+2r \cos(\pi u)]^2} \]
- Asymmetric in sense of dynamics. For \( r = 0 \) becomes potential of standard Frenkel - Kontorova model.
Asymmetric deformable substrate potential

- Asymmetric deformable potential is more realistic.
Asymmetric deformable substrate potential is more realistic.
For integer value of $\omega$:
For integer value of $\omega$:
Farey construction
Farey construction

- Farey construction
- \( r = 0.01 \)
- Farey construction
- \( r = 0.01 \)
Thank you!